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**A SIMPLE EQUATION FOR CORRELATING TURBULENT  
HEAT TRANSFER TO A GAS**

by R. J. Simoneau and R. C. Hendricks  
Lewis Research Center  
Cleveland, Ohio

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TECHNICAL MEMORANDUM

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ABSTRACT

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Many investigators have employed the conventional Dittus-Boelter (Nusselt type) equation modified by a wall to bulk temperature ratio to correlate turbulent heat-transfer data for gases flowing through heated tubes. It has been found that the reported convective data for hydrogen, helium, air, and carbon dioxide can be correlated to the same accuracy as the Nusselt type correlations by use of the equation

$$h = K(\rho V)_b^{0.8} d^{-0.2} \sqrt{T_b/T_w}$$

where  $K$  is a constant for any given gas. This equation suggests that for the four gases investigated, a convective correlation for a given gas need not include the temperature dependence of the thermal and transport properties.

Based on the plots of these reported data, the values of  $K$  are: for hydrogen, 0.0480, for helium, 0.0200, for air, 0.00420, and for carbon dioxide, 0.00385.

These data covered the conditions of film temperature from 500° to 3400° R with wall to bulk temperature ratios from 1.1 to 9.9. No appreciable dissociation was reported in the data. The minimum bulk temperature was 150° above the critical temperature. The bulk Reynolds number ranged from 5000 to 1,500,000 with heat fluxes up to 3 Btu per second per square inch. Tube diameters varied from 1/8 to 1/2 inch.

author  
X-52011

## INTRODUCTION

The question of how temperature-dependent thermal and transport properties influence turbulent heat-transfer correlations has been investigated extensively in recent years (refs. 1 to 8). Much of this work has been directed toward determining the temperature at which to evaluate these properties. A large fluid to wall temperature gradient, similar to that experienced in rocket cooling channels, will cause a correspondingly large change in thermal properties across the boundary layer. For this reason investigators have felt that the technique of accounting for these properties should have a marked influence on the heat-transfer coefficients.

Bartz (ref. 9), Wolf (ref. 6), and others have suggested that turbulent heat transfer is dominated by the mass flow rate. In reference 9, the relation suggested is

$$h \propto (\rho V)_b^m \quad (1)$$

(Symbols are defined in the appendix.) A similar approach is presented in reference 6 in a plot of  $h$  as a function of  $(\rho V)_b$  in figure 37 of that reference. The nature of this proportionality is explored herein. The approach will depart from the conventional approach of assuming that this proportionality is best expressed in terms of the standard Nusselt type equation. The dominance of mass velocity suggests that in correlations based only on measurements of heat flux, mass flow rate, bulk temperature, tube diameter, and temperature difference, it may not be necessary to account for thermal- and transport-property variations across the boundary layer. To determine this, the measured

parametric data of some recent heated tube experiments (refs. 1 to 8) were reexamined in this report.

References 1 to 8 represent a series of heated tube investigations of turbulent heat transfer to gases at moderate to large values of  $\Delta T$ . In the regions covered by these experiments the conventional Dittus-Boelter (Nusselt type) equation, modified by the wall to bulk temperature ratio, has successfully correlated the data. In many cases the properties were evaluated at film temperature; however, some investigators reported correlations based on bulk temperatures. The question of which temperature to use to evaluate the properties has not been fully resolved. This correlation usually takes on one of the two following forms:

$$Nu_b = C Re_b^{0.8} Pr_b^{0.4} (T_w/T_b)^a \quad (2)$$

$$\begin{aligned} Nu_f &= C Re_{mod}^{0.8} Pr_f^{0.4} \\ &= C Re_f^{0.8} Pr_f^{0.4} (T_f/T_b)^{1/4} \end{aligned} \quad (3)$$

For the investigations referenced,  $a = -0.50$  to  $-0.55$  and  $b = -0.80$ . The effectiveness of a correlation based on the measured parameters alone will be determined by comparing it with the reported correlations of the same data using equations (2) and (3).

The data considered for analysis from the references 1 to 8 included all that used by the individual authors for their correlations except the data of references 2, 7, and 8. All the runs from reference 2 were used, but only one local station was analyzed. The data of reference 7 were omitted because they yielded high heat-transfer coef-

ficients. In reference 8 the authors reported that the data of the earlier report, reference 7, yielded high values of  $h$  and would not be included in their correlations. The data used for correlation in reference 8 was not presented in that report. References 2, 3, and 5 also did not present the measured data, therefore, it was obtained by direct communication with the authors. Only the gas data of reference 4 was used for correlation. The data analyzed covered the overall conditions of film temperature from  $500^{\circ}$  to  $3400^{\circ}$  R with wall to bulk temperature ratios from 1.1 to 9.9. No appreciable dissociation was reported in the data. The minimum bulk temperature was  $150^{\circ}$  above the critical temperature. The bulk Reynolds number ranged from 5000 to 1,500,000 with heat fluxes up to 3 Btu per second per sq in. Tube diameters varied from  $1/8$  to  $1/2$  inch. A more detailed list of the data domains is given in table I.

#### CORRELATION TECHNIQUE

In the heated tube experiments, such as those of references 1 to 6 heat flux  $q$ , mass flow rate  $\dot{w}$ , bulk temperature  $T_b$ , and tube diameter  $d$ , are the control parameters. The only other measured parameter, wall temperature  $T_w$  or  $\Delta T$ , is dependent on the control parameters. If a correlation is to be based on these measured parameters, it then can be expressed in the form

$$\Delta T = f(q, \dot{w}, T_b, d) \quad (4)$$

Pressure is not included in equation (4) because there were no apparent pressure-level effects in the analysis presented herein. The hydrogen data of reference 1, are plotted in figure 1 in order to evaluate equation (4). Figure 1 is a plot of the relation

$$\frac{q}{\Delta T} = h \propto \left(\frac{\omega}{A}\right)^{0.8} d^{-0.2} \quad (5)$$

A dependence of  $h$  on  $T_w/T_b$  is evident in figure 1 for  $T_w/T_b > 3$  and also has been observed and reported in the references 1 to 6. Data from each reference were plotted separately in the same manner as indicated in figure 1 to determine the dependence of  $h$  on the wall to bulk temperature ratio. A plot of  $h$  against  $T_w/T_b$  at a given value of  $(\omega/A)^{0.8} d^{-0.2}$  for references 1 to 6 is shown in figure 2. The heat-transfer coefficient  $h$  varies as  $(T_w/T_b)^a$ , where  $a$  can be selected as approximately -0.5. One explanation for the spread and varying slope of figure 2 could be effects of the tube length to diameter ratio ( $L/D$ ). It was decided to leave the  $L/D$  correction out of this analysis for simplicity.

In figure 3, the abscissa of figure 1 was modified by multiplying the right side of equation (5) by  $\sqrt{T_b/T_w}$ . Various values of  $a$  between -0.4 and -0.6 were investigated and appeared to make no significant change in the spread of the data. Figure 3 represents data from references 1 to 6 plotted with this modification by the wall to bulk temperature ratio included. Some of the investigators of references 1 to 6 reported average heat-transfer coefficients and other reported local values. No effort was made to distinguish between them in figure 3. There were no apparent differences. Inspection of figure 3 yields the equation

$$h = K(\rho V)_b^{0.8} d^{-0.2} \sqrt{T_b/T_w} \quad (6)$$

where  $K$  appears to be a constant unique to each gas. The values of  $K$  for each of the gases investigated are listed in table II. These values were obtained from the lines drawn through the data of figure 3. Equation (6) correlates all the data within  $\pm 12$  percent as shown in table II, column 4.

#### COMPARISON TO NUSSELT TYPE CORRELATION

##### Accuracy

An indication of the comparative correlation accuracies of equations (2) and (6) may be obtained by inspection of table II. Column 1 represents the deviation of the data as reported by the authors of references 1 to 6. These authors used either equation (2) or (3) to correlate their data; therefore, the deviations in column 1 represent the accuracy with which a Nusselt type equation, such as equation (2), will correlate the reported data. Column 4 represents the accuracy with which equation (6) will correlate the same data, the data of figure 3. Inspection of the hydrogen data, for example, indicates a deviation of  $\pm 11$  percent using equation (6), which compares very favorably with an average of  $\pm 10$  percent using the Nusselt correlation. Inspection of figure 3 shows that the spread of the data, column 2 of table II, could be decreased and the data would be better fit by using a slope other than  $m = 0.8$ . This has been observed by others, and  $m = 0.8$  is retained for convention and so that equation (6) can be compared with the reported Nusselt correlations.



### Relation With Nusselt Equation

The terms in equation (2) can be regrouped in the following manner:

$$h = C \left( \frac{k^{0.6} c_p^{0.4}}{\mu^{0.4}} \right)_x (\rho V)_b^{0.8} d^{-0.2} (T_w/T_b)^a \quad (7)$$

The subscript  $x$  on the property grouping indicates equations (2) and (7) can represent correlations based on either bulk, film, or wall temperature properties. The form will remain the same, but the constant  $C$ , or the exponent  $a$ , or both will be different for different reference conditions (ref. 1). When the Nusselt correlation (eq. (2)), is written in the manner of equation (7), it is of the same form as equation (6). This should be expected since both correlations are based on the same measured parameters. Inasmuch as equations (6) and (7) will correlate the data to within the same accuracy, a comparison of the two equations can be made to yield the relation

$$K = C\phi_x = \text{const} \quad (8)$$

where

$$\phi = \frac{k^{0.6} c_p^{0.4}}{\mu^{0.4}} \quad (9)$$

Figure 4 indicates that, with the exception of para-hydrogen, for perfect and near perfect gases,  $\phi$  is a monotonically increasing function of temperature. Since figure 3 indicates that  $K$  is a constant for each gas, equation (8) requires that  $C$ , the conventional Dittus-Boelter coefficient, be a function of temperature that decreases in inverse proportion to  $\phi$ . Figure 5 is a plot of  $C$  against  $T$  for all the hydrogen data of reference 1. These  $C$  values were computed for bulk, film, and wall

reference temperatures for each run using equation (7) with a value of  $a = -0.5$ . The dashed line represents  $\phi$  against  $T$  for normal hydrogen using the property data equations presented in reference 1. The solid line represents  $C$  against  $T$  based on equation (8), specifically,

$$C(T) = \frac{K}{\phi(T)} = \frac{0.0480}{\phi(T)_{N-H_2}} \quad (10)$$

The average deviation of the data about this curve is +14 to -11 percent. This average deviation for the  $C$  value of each run about the curve of equation (10) is also the average deviation about  $K = 0.0480$  for the data of reference 1. It is consistent with that reported in column 3, of table II for reference 1.

Inspection of column 3 of table II indicates that the data of each investigator shown in figure 3 do not fall symmetrically about the lines described by equation (6). This nonsymmetry was inspected for film or bulk temperature effect, which might suggest  $K$  is also a function of temperature. No such effect was apparent in figure 3. The data seemed to fall in a random pattern about the mean line without regard to temperature. Figure 5, however, suggests a slight bulk-temperature effect on  $K$ . The temperature effect does not appear to be strong; however,  $K$  seems to be an increasing function with temperature. From the available data, it is not possible to predict a more accurate  $K$  as a function of temperature than one gets by assuming it constant. An inspection of some of the near-critical data of reference 4 also indicates that  $K$  decreases slightly as the bulk temperature approaches the critical value.

### Extrapolation to Other Gases

Another question of concern is how well can one go from one gas to another or extrapolate to unknown gases using equation (6) compared with the Nusselt equation. Assume that, knowing  $K$  for hydrogen to be 0.0480, one wishes to find the  $K$  values for helium, air, and carbon dioxide. A reference temperature at which to perform the calculations can be selected from figure 4. This point should be sufficiently removed from the critical temperature to ensure that all the gases are behaving in the same manner, that is, like perfect gases. For these gases, a selection of  $800^{\circ} - 1000^{\circ} \text{ F}$  would seem reasonable. From equation (8) a Dittus-Boelter type coefficient can be evaluated for normal hydrogen at some reference temperature in the following manner:

$$C_{1000^{\circ} \text{ R}} = \frac{K_{\text{N-H}_2}}{\phi_{\text{N-H}_2, 1000^{\circ} \text{ R}}} = 0.0213 \quad (11)$$

Applying equations (8) and (11) and the data of figure 4 yields the estimates of  $K$  given in the following table:

Gas	Experi- mental, from fig. 3	Calculated at $800^{\circ} \text{ F}$ reference temperature; $C(T) = 0.0223$ from eq. (11)		Calculated at $1000^{\circ} \text{ F}$ reference temperature; $C(T) = 0.0213$ from eq. (11)	
	Propor- tionality constant, $K$	Propor- tionality constant, $K$	Error percent	Propor- tionality constant, $K$	Error, percent
Helium	0.0200	0.0196	-2.0	0.0191	-4.5
Air	.00420	.00381	-9.3	.00383	-8.8
Carbon dioxide	.00385	.00348	-9.6	.00381	-1.0

Thus it can be seen that, for the gases investigated, it is possible to estimate with considerable accuracy the  $K$  of a gas based on the experimentally known  $K$  of another gas. These results tend to support a similar extrapolation to gases for which  $K$  is not experimentally known. At present, however, this extrapolation must be limited to gases whose curves of  $\phi$  against  $T$  are of the same general form as those of figure 4.

#### CONCLUSIONS

For the gases examined, and probably for any near-perfect gas, equation (6) with a specific constant  $K$  for each gas will correlate turbulent heat-transfer data successfully within currently acceptable accuracy limits. Equation (6) is very simple in form and should be particularly attractive in design-type parametric studies.

It is not to be inferred from this study that the properties do not influence the actual heat-transfer mechanism. All that is being pointed out is that property variations of a given gas do not influence a data correlation based on measurements of heat flux, mass flow rate, bulk temperature, tube diameter, and temperature difference. More sophisticated experiments are needed to assess the influence of property variations on heat transfer over a range of temperatures. These experiments would have to be designed to assess the relation that the basic fluid-flow and heat-transfer mechanism have with the thermal and transport properties. Apparently heat flux, mass flow rate, and temperature difference reflect these relations such that they can be correlated effectively independent of properties. To determine why this is so would seem to be a good experimental and theoretical starting point.

APPENDIX - SYMBOLS

A	area, $\text{ft}^2$
a,b	exponents
C	Dittus-Boelter coefficient, dimensionless
$c_p$	specific heat, $\text{Btu}/(\text{lb}_{\text{mass}})(^{\circ}\text{R})$
d	tube diameter, ft
h	heat-transfer coefficient, $\text{Btu}/(\text{hr})(\text{ft}^2)(^{\circ}\text{R})/(\text{ft})$
K	proportionality constant, $\text{Btu}/(^{\circ}\text{R})(\text{lb}_{\text{mass}}^{0.8})(\text{hr}^{0.2})(\text{ft}^{0.2})$
k	thermal conductivity, $\text{Btu}/(\text{hr})(\text{ft}^2)(^{\circ}\text{R})(\text{ft})$
L/D	ratio of tube length to tube diameter, dimensionless
m	exponent
Nu	Nusselt number, $hd/k$ , dimensionless
Pr	Prandtl number, $\mu c_p/k$ , dimensionless
q	heat flux, $\text{Btu}/(\text{hr})(\text{ft}^2)$
Re	Reynolds number, $(\rho V)_b d/\mu_x$ , dimensionless
$\text{Re}_{\text{mod}}$	modified Reynolds number, $\frac{\rho_f V_b d}{\mu_f}$ , dimensionless
T	temperature, $^{\circ}\text{R}$
$\Delta T$	temperature difference, $T_w - T_b$ , $^{\circ}\text{R}$
$\rho V$	mass velocity, $\text{lb}_{\text{mass}}/(\text{hr})(\text{ft}^2)$
$\mu$	viscosity, $\text{lb}_{\text{mass}}/(\text{ft})(\text{hr})$
$\phi$	property grouping, $k^{0.6} c_p^{0.4}/\mu^{0.4}$ , $\text{Btu}/(^{\circ}\text{R})(\text{lb}_{\text{mass}}^{0.8})(\text{hr}^{0.2})(\text{ft}^{0.2})$
$\dot{m}$	mass flow rate, $\text{lb}_{\text{mass}}/\text{hr}$

Subscripts:

b	bulk temperature
f	film temperature

w wall temperature

x reference temperature

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TABLE I. - DATA USED FROM INVESTIGATIONS OF REFERENCES 1 TO 8

Gas	Film temperature, °R	Wall to bulk temperature, ratio, $T_w/T_b$	Pressure, lb/sq in gage	Exponents		Dittus-Boelter coefficient, C	References
				a	b		
Hydrogen, helium	500 - 1350	1.5 - 9.9	30 - 1350	<sup>a</sup> -0.55	-----	<sup>a</sup> 0.025	1
Hydrogen, helium	800 - 3400	1.4 - 6.6	40 - 100	-----	-0.80	.021	2
Hydrogen, helium	500 - 1500	1.2 - 3.0	<sup>b</sup> 200 - 1000	-----	-.80	.021	3
Hydrogen	570 - 900	1.1 - 1.7	215 - 640	-----	-----	-----	4
Air	580 - 1700	1.1 - 2.5	15 - 65	-----	-.80	<sup>c</sup> .023	5
Air, carbon dioxide	800 - 1300	1.6 - 2.8	100	-----	-.80	<sup>d</sup> .037	6
Nitrogen, helium, carbon dioxide	350 - 2000	1.2 - 1.6	up to 1000	-.50	-----	<sup>e</sup> .036	7, 8

<sup>a</sup>More than one value of a and C was used in ref. 1.

<sup>b</sup>Information received in a private communication.

<sup>c</sup>Wall temperature correlation was used instead of film correlation.

<sup>d</sup>Wall temperature correlation and L/D correction were used.

<sup>e</sup>Tube length to diameter ratio correction was used.

TABLE II. - COMPARISON OF EQUATION (6) WITH NUSSELT CORRELATION

Gas	Proportion- ality constant, K	Average Deviation, percent				Refer- ence
		1- (a)	2 (b)	3 (c)	4 (d)	
Hydrogen	0.0480	<sup>a</sup> ±8	±12	+13 to -11		1
		±10	±12	+18 to -6	+11 to	2
		<sup>f</sup> ±5	±7	+5 to -9	-11	3
		---	±15	+9 to -20		4
Helium	0.0200	±8	±9	+12 to -5	+12 to	1
		±10	±13	+17 to -8	-10	2
		<sup>f</sup> ±8	±10	+7 to -12		3
		±12	(e)	(e)	(e)	7,8
Air	0.00420	---	±14	+16 to -12	+11 to	5
		<sup>f</sup> ±7	±8	+8 to -8	-10	6
Carbon dioxide	0.00385	<sup>f</sup> ±7	±9	+8 to -9	+8 to	6
	(e)	±12	(e)	(e)	-9 (e)	7,8
Nitrogen	(e)	±12	(e)	(e)	(e)	7,8

<sup>a</sup>Deviation as reported in the references based on eq. (2) or (3).

<sup>b</sup>Spread of the data of each reference plotted as in fig. 3.

<sup>c</sup>Deviation for the data of each reference from the line described by eq. (6).

<sup>d</sup>Deviation for all the data of a given gas from the line described by eq. (6).

<sup>e</sup>Insufficient data published to determine K or to make deviation analysis. These data appear to support the rest in about the same manner; however, they seem to indicate higher K values than other reported data. (See INTRODUCTION).

<sup>f</sup>Estimated from published curves.

# EFFECT OF MASS VELOCITY AND TUBE GEOMETRY ON HEAT-TRANSFER COEFFICIENT

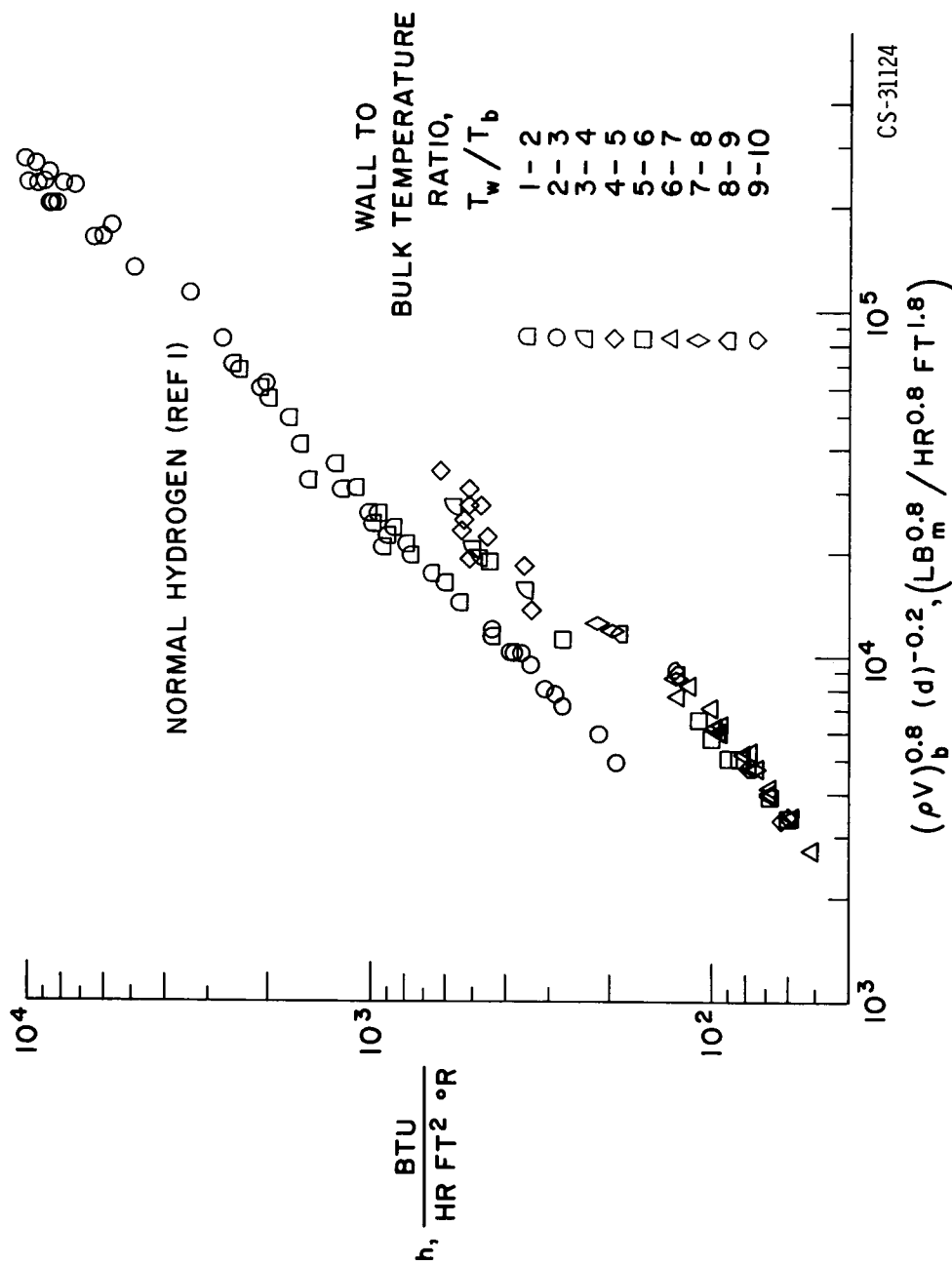


FIGURE 1.

# EFFECT OF WALL-TO-BULK TEMPERATURE RATIO ON HEAT-TRANSFER COEFFICIENT

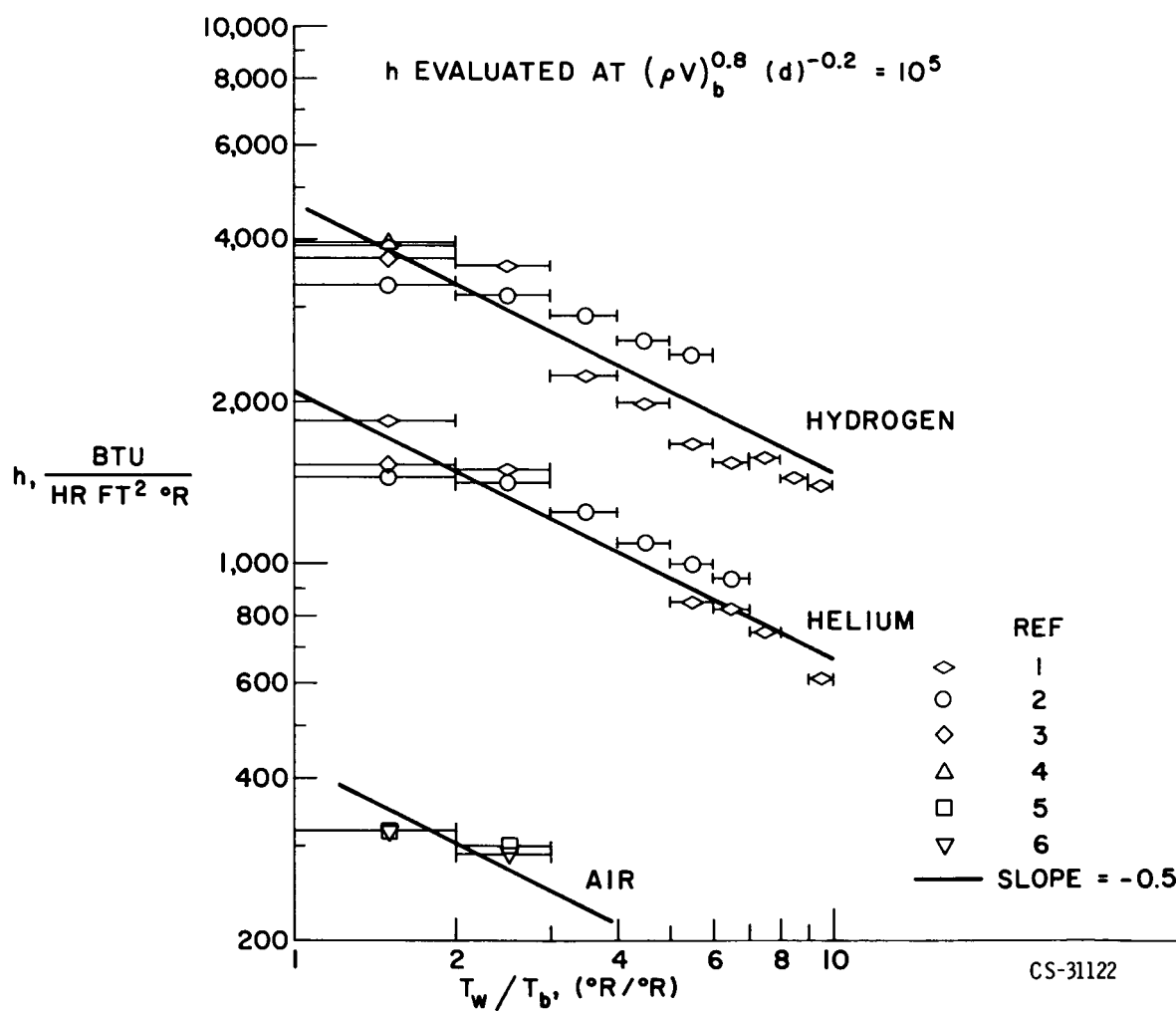


FIGURE 2.

# EFFECT OF MASS VELOCITY, TUBE GEOMETRY, AND WALL TO BULK TEMPERATURE RATIO ON HEAT-TRANSFER COEFFICIENT

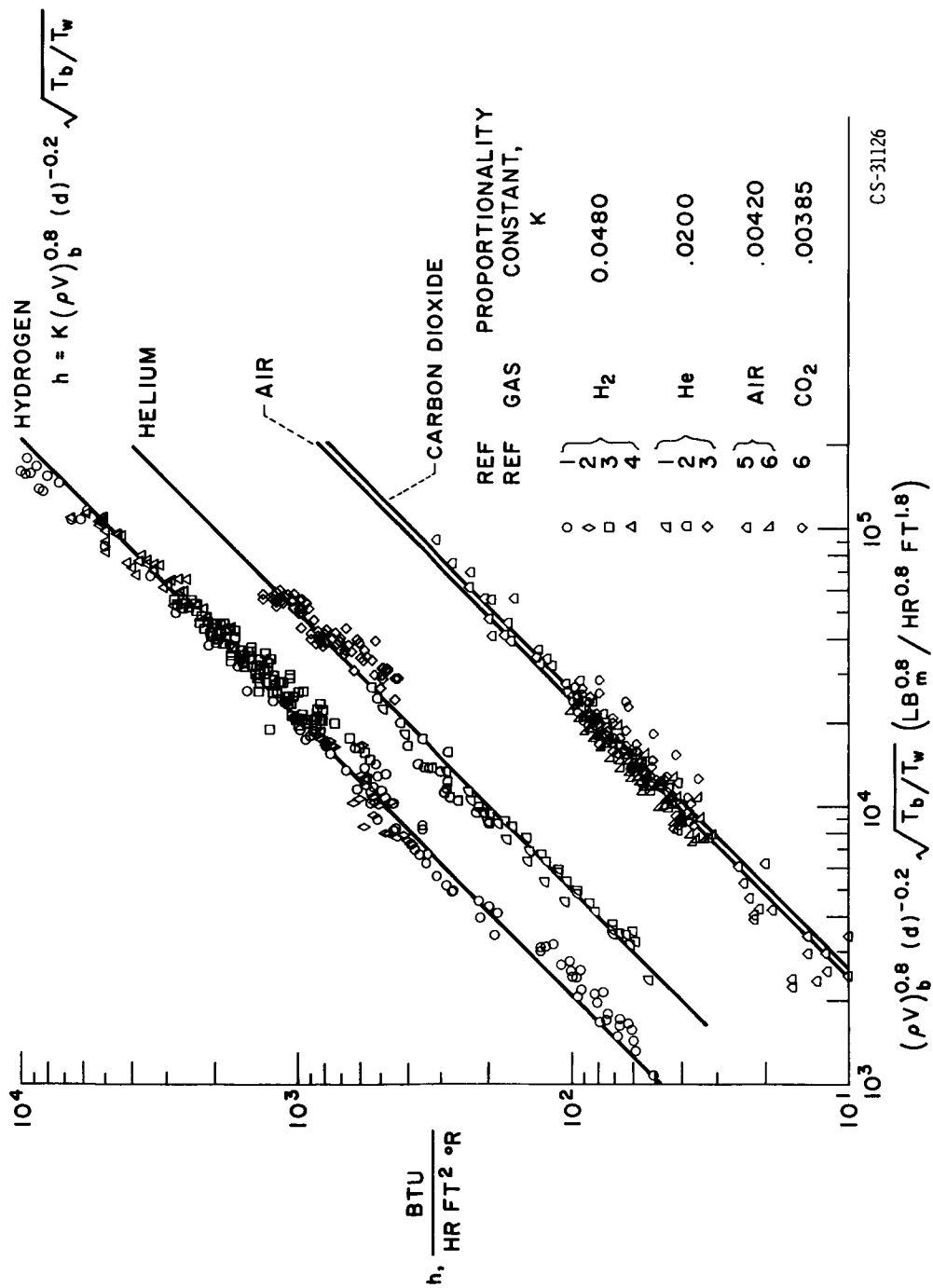


FIGURE 3.

# EFFECT OF TEMPERATURE ON PROPERTY GROUP

$$(\phi = k^{0.6} c_p^{0.4} / \mu^{0.4})$$

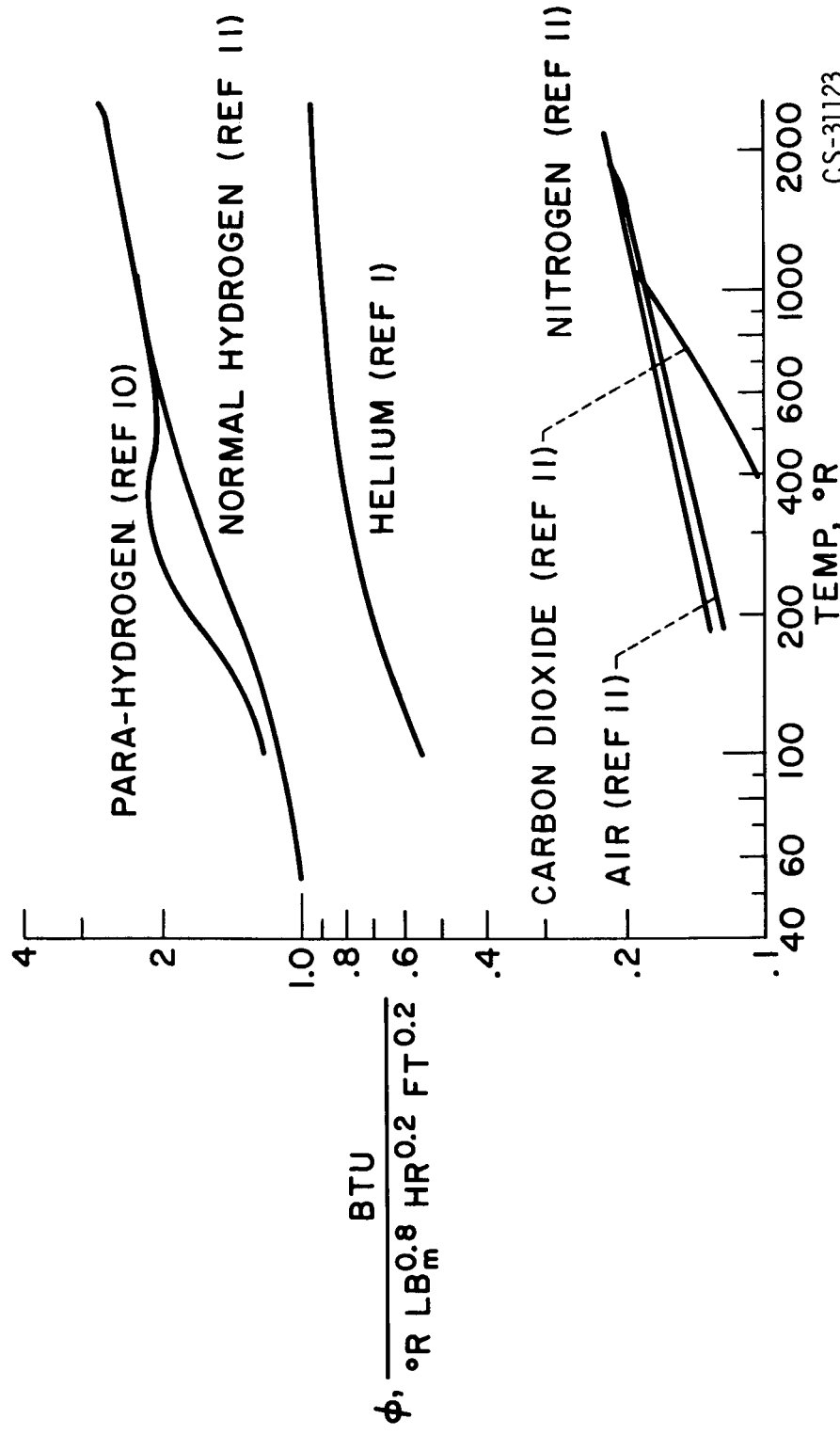


FIGURE 4.

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# EFFECT OF TEMPERATURE ON DITTUS-BOELTER COEFFICIENT

NORMAL HYDROGEN (REF 1)

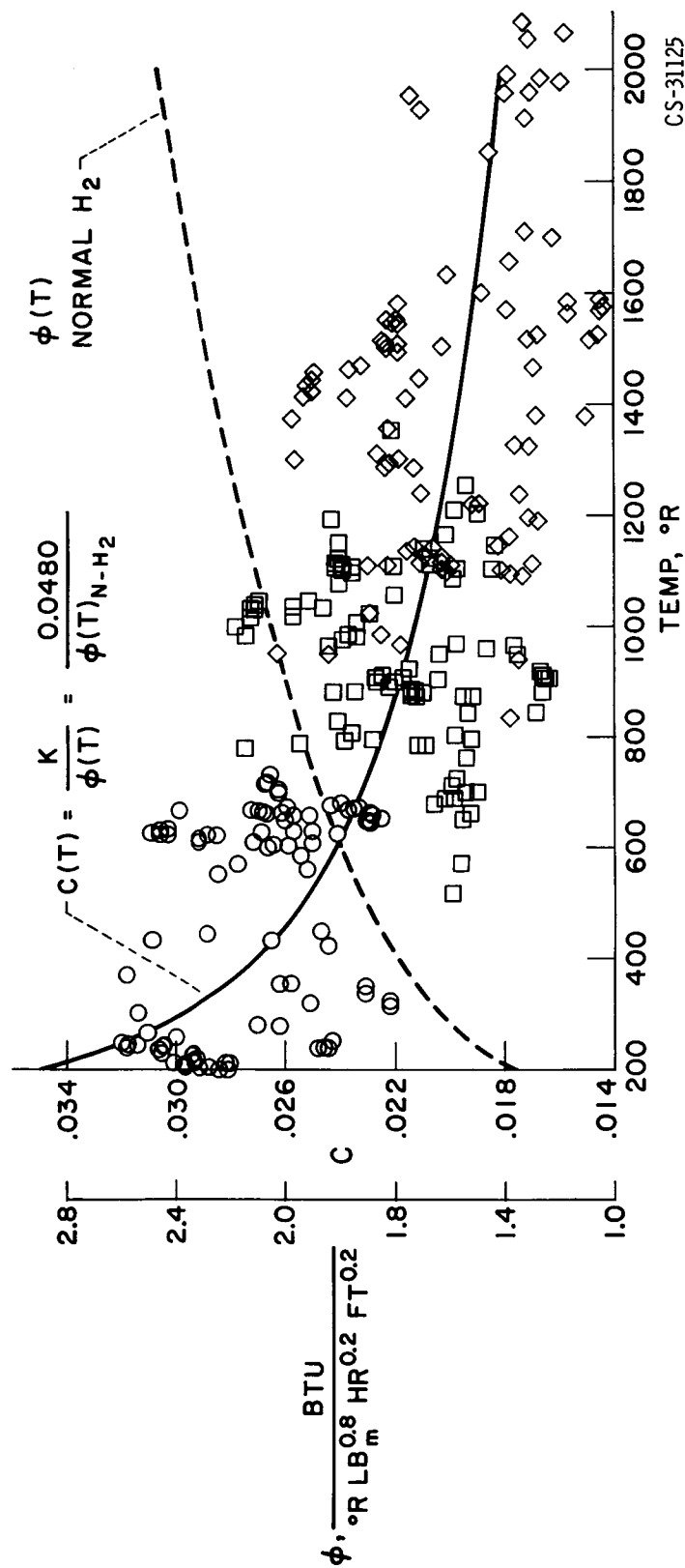


FIGURE 5.

CS-31125